Heating and Evaporation of Turbulent Falling Liquid Films

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Falling film heat exchangers find wide application in industry. In this work an analysis of heat transfer in turbulent falling films is presented. The objective is to develop a single analytical formula for the fully developed heat transfer coefficient for the cases of evaporation (or condensation) and heating (or cooling). In a recent paper, Sandall et al. (1984), using a large *Pr* asymptotic approximation procedure, derived an analytical expression for the developed heat transfer coefficient in turbulent falling liquid films. This equation was shown to be in excellent agreement with the evaporation heat transfer data of Chun and Seban (1971); however, it is found that the theoretical formula significantly overpredicts the data of Wilke (1962) for heating of turbulent films. Specifically, we find that our theoretical formula (Sandall et al.) predicts the Chun and Seban evaporation data to within an absolute mean deviation of 3.1%, but when the formula is modified to apply to heating or cooling it predicts the Wilke heating data with an absolute mean deviation of 67.3%. In this note an analysis is carried out in which we somewhat modify the eddy diffusivity that was used to derive our prior results (Sandall et al., 1984). Here we use the eddy diffusivity for the interface region from Hubbard et al., (1976). The eddy diffusivity in the wall region is taken to be the same as was used to derive our prior results except that the Limberg (1973) reduction factor is applied, as recommended by Seban and Faghri (1976). The resulting analytical expression for the heat transfer coefficient is found to be capable of representing both the evaporation data of Chun and Seban and the heating data of Wilke.

The evaporation or condensation problem addressed in this research has been studied theoretically by Seban (1954), Dukler (1960), Lee (1964), Kunz and Yerazunis (1969), Mills and Chung (1973), Seban and Faghri (1976), Hubbard et al. (1976), Mostofizadeh and Stephan (1981), Kutateladze (1982), and Sandall et al. (1984). Heating or cooling of a turbulent liquid film has been studied by Limberg (1973) and Carey (1985). Both heating and evaporation have been treated by Seban and Faghri (1976), Seban (1978), and Yih and Liu (1983).

In the model developed here the turbulent transport near the wall is described by a modified Van Driest equation for the eddy diffusivity as proposed by Hanna et al., (1981). This expression was further modified here for turbulent transport in falling films using the reduction factor suggested by Limberg (1973) and recommended by Seban (1978). The resulting eddy diffusivity expression is similar to the Nikuradse distribution (Seban and Faghri, 1976).

The eddy diffusivity expression proposed by Hubbard et al. (1976) is used to model the turbulent transport very near the free surface. The formula of Hubbard et al. was derived from gas absorption experiments with turbulent falling films over a wide range of liquid properties with and without interfacial shear forces. Since Hubbard et al. used the Brotz (1954) relationship for the film thickness in deriving their correlation for the interface diffusivity, we have also used the Brotz formula for consistency.

At large Prandtl numbers, the major resistances to heat transfer reside near the transport boundaries. For evaporation this will be near the wall and the free surface. For heating, the significant resistance is near the wall, since in this case there is no transport at the interface. Therefore, it is not necessary to precisely describe the eddy diffusivity in the central region of the film in order to calculate the heat transfer. Seban and Faghri (1976) and Seban (1978) have investigated the suitability of various eddy diffusivity models for describing evaporative heat transfer across turbulent falling films. The general approach used in this work is consistent with their recommendations.

Theory

The governing energy equation for heat transfer across a turbulent falling film is

$$u\frac{\partial T}{\partial x} = \frac{\partial}{\partial y}\left[(\alpha_T + \epsilon_H)\frac{\partial T}{\partial y}\right] \tag{1}$$

In this analysis we assume that there is no nucleation within the liquid film. In accordance with earlier work (Sandall, et al. 1984), and in order to keep the analytical development tractable, the turbulent Prandtl number is taken to be constant at a value of 0.9.

In the case of evaporation, for thermally developed conditions (large x), $\partial T/\partial x = 0$, and Eq. 1 can be integrated subject to the boundary conditions $y = 0$, $\partial T/\partial y = -q_0/k$, and $y = \delta$, $T = T_{sat}$ to give

$$\frac{h}{k}\left(\frac{\nu^2}{g}\right)^{1/3} = \frac{Pr\,\delta^{+1/3}}{\int_0^{\delta^+}\left(\frac{1}{Pr}+\frac{\epsilon^+}{Pr_t}\right)dy^+} \tag{2}$$

In the case of heating with a constant surface temperature boundary condition, it is known that for Prandtl numbers greater than about unity, the heat transfer coefficient is nearly the same as that for constant flux. For fully developed heat transfer with a constant wall flux, $\partial T/\partial x = dT_m/dx =$ constant. By determining this constant from an overall energy balance, together with the usual definitions of the Reynolds number and friction velocity (see Notation) and noting that for heating at Pr larger than about unity the interface temperature is approximately equal to the bulk temperature, Eq. 1 can be integrated again to give Eq. 2.

Thus the problem of calculating the heat transfer coefficient, for either heating or evaporation, reduces to the evaluation of the integral in Eq. 2. In order to do this, we split this integral according to the most important regions for heat transfer resistance as

$$\int_0^{\delta^+} f(y^+)\,dy^+ = \int_0^{M_1} f(y^+)\,dy^+ + \int_{M_1}^{M_2} f(y^+)\,dy^+ + \int_{M_2}^{\delta^+} f(y^+)\,dy^+ \tag{3}$$

where $f(y^+) = (1/Pr + \epsilon^+/Pr_t)^{-1}$. The three integrals on the righthand side of Eq. 3 represent the wall, the core, and the interface regions, respectively. It must be emphasized that the interface resistance contribution of Eq. 3 is applicable only in the case of evaporation; for the case of heating this resistance is zero and Eq. 3 would contain only the integrals from 0 to M_1 and from M_1 to δ^+, representing the wall and core resistances, respectively.

We now wish to evaluate the integrals in Eq. 3 asymptotically for large Pr. Past experience shows that such a procedure often produces a simple analytical representation that is valid for a wide range in Pr. In the region near the wall, the maximum contribution comes from near $y^+ = 0$; thus in this region the eddy diffusivity is expressed in a Taylor series expansion. But, as suggested by Limberg (1973) and Seban and Faghri (1976), this eddy diffusivity is multiplied by a reduction factor to give

$$\epsilon^+ = [K_3(y^+)^3 + K_4(y^+)^4 + K_5(y^+)^5 + \cdots]\cdot[\exp(-1.66\,y^+/\delta^+)]^2 \tag{4}$$

Expanding the reduction factor in a Taylor series about $y^+ \rightarrow 0$ and using the Brotz relationship ($\delta^+ = 0.0174\,Re$) to solve for δ^+ in terms of the Reynolds number, we obtain

$$[\exp(-1.66\,y^+/\delta^+)]^2 = 1 + V_1 y^+ + V_2 y^{+2} + \cdots \tag{5}$$

where

$$V_1 = -189.7\,Re^{-1} \quad V_2 = (3.599 \times 10^4)\,Re^{-2} \tag{6}$$

For the modified Van Driest eddy diffusivity (Hanna, et al. 1981), to be used here, the K_i coefficients are

$$K_3 = 9.1033 \times 10^{-4},\ K_4 = 8.3330 \times 10^{-5},\ K_5 = 1.3621 \times 10^{-6} \tag{7}$$

Using the K_i values from Eq. 7, and substituting Eqs. 5 and 6 into Eq. 4, we obtain

$$\epsilon^+ = K_{f3}(y^+)^3 + K_{f4}(y^+)^4 + K_{f5}(y^+)^5 + \cdots \tag{8}$$

where

$$K_{f3} = 9.1033 \times 10^{-4} \tag{9}$$

$$K_{f4} = 8.3330 \times 10^{-5} - 0.173\,Re^{-1} \tag{10}$$

$$K_{f5} = 1.3621 \times 10^{-6} - 1.58 \times 10^{-2}\,Re^{-1} + 32.8\,Re^{-2} \tag{11}$$

With this expression for the eddy diffusivity near the wall, the first integral on the righthand side of Eq. 3 can be evaluated asymptotically for high Prandtl numbers (Sandall, et al., 1984) to give

$$\int_0^{M_1} f(y^+)\,dy^+ = C_1 Pr^{2/3} + C_2 Pr^{1/3} + C_3 \ln Pr + C_4(M_1) + 0\left(\frac{1}{Pr^{1/3}}\right) \tag{12}$$

where

$$C_1 = 12.05 \quad C_2 = -7.32 + 1.52 \times 10^4\,Re^{-1} \tag{13}$$

$$C_3 = 2.27 - 4.58 \times 10^3\,Re^{-1} + 1.14 \times 10^3\,Re^{-2} \tag{14}$$

The two remaining integrals for the core and interface regions, evaluated in a similar way, are

$$\int_{M_1}^{M_2} f(y^+)\,dy^+ = 0(1) + 0\left(\frac{1}{Pr}\right) \tag{15}$$

and

$$\int_{M_2}^{\delta^+} f(y^+)\,dy^+ = B_1 Pr^{1/2} + B_2 \ln Pr + 0(1) + 0\left(\frac{1}{Pr^{1/2}}\right) \tag{16}$$

In the integral of Eq. 16, the eddy diffusivity model of Hubbard et al. (1976) for the interface region is used. This expression is

$$\epsilon^+ = \frac{8.13 \times 10^7 (\nu g)^{2/3}}{Ka u^{*2}}\,Re^{2n}\left[1 + b\left(\frac{\tau_i}{\tau_w}\right)\right]^2 (\delta^+ - y^+)^2 \tag{17}$$

For the case of zero interfacial shear considered here ($\tau_i/\tau_w \simeq 0$) and using the Brotz relationship for u^* we get

$$\epsilon^+ = (1.21 \times 10^{-15}) Ka^{-1}\,Re^{(2n-2/3)}(\delta^+ - y^+)^2 \tag{18}$$

where

$$n = 6.95 \times 10^2 (\nu)^{1/2} (\nu \text{ in m}^2/\text{s}) \tag{19}$$

Equations 18 and 19 give B_1 as

$$B_1 = (2.875 \times 10^7) Ka^{1/2} Re^{(1/3 - n)} \tag{20}$$

Due to a lack of information, B_2 was taken to be zero.

As discussed by Sandall et al. (1984), the contributions of 0(1) in Eqs. 12, 15, and 16 are all independent of Prandtl number but depend on Reynolds number. Since these contributions cannot be calculated analytically, a simple empirical function of the form $f(Re) = P + Q \ln Re$ was assumed.

Finally, substitution of Eqs. 12, 15, and 16 into Eq. 2 yields the expression for the heat transfer coefficient as

$$\frac{h}{k}\left(\frac{\nu^2}{g}\right)^{1/3} = \frac{0.2596 Pr Re^{1/3}}{C_1 Pr^{2/3} + C_2 Pr^{1/3} + C_3 \ln Pr + B_1 Pr^{1/2} + P + Q \ln Re} \tag{21}$$

where Eqs. 13, 14, and 20 are used. For the problem of evaporation, Eq. 20 is used as it stands. For the problem of heating, the quantity B_1 is put equal to zero, since in this case there is no resistance at the interface.

Comparison with Experiment

The values of the constants P and Q are determined by fitting Eq. 21 simultaneously to both the experimental data of Wilke (1962) for heating, and of Chun and Seban (1971) for evaporation, using least squares. There are 26 data points for heating and 29 for evaporation. The values of P and Q determined by least squares are found to be $P = 37.78$ and $Q = -1.970$. The corresponding mean deviation between experimental and predicted values is 6.1%. A graphical comparison of the results is shown in Figure 1. Figures 2 and 3 show the variation of the dimensionless heat transfer coefficient with Reynolds number. Figure 2 shows a comparison of the Wilke heating data with the predictions of Eq. 21. These data agree with Eq. 21 with a mean deviation of 3.6%. Figure 3 compares the Chun and Seban evaporation data with Eq. 21. In this case the data agree to within a mean deviation of 8.3%. Considering past attempts to correlate both the heating and evaporation data, the new correlation developed here seems quite reasonable.

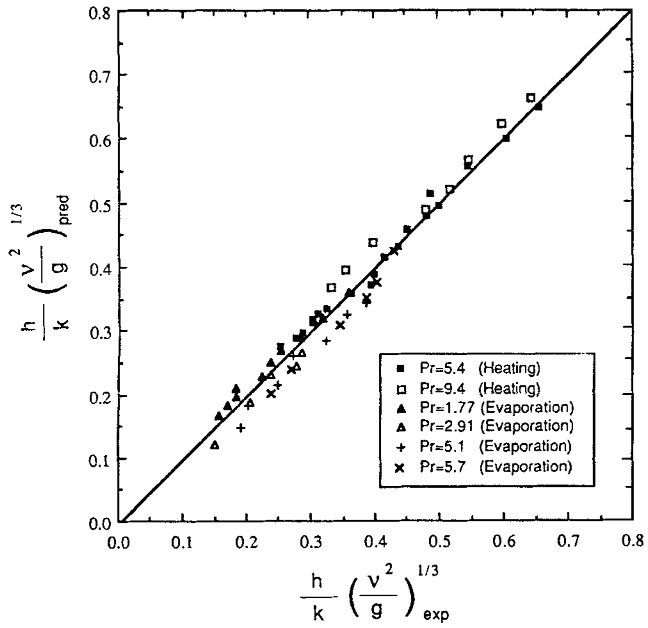

Figure 1. Experimental and predicted heat transfer coefficients.

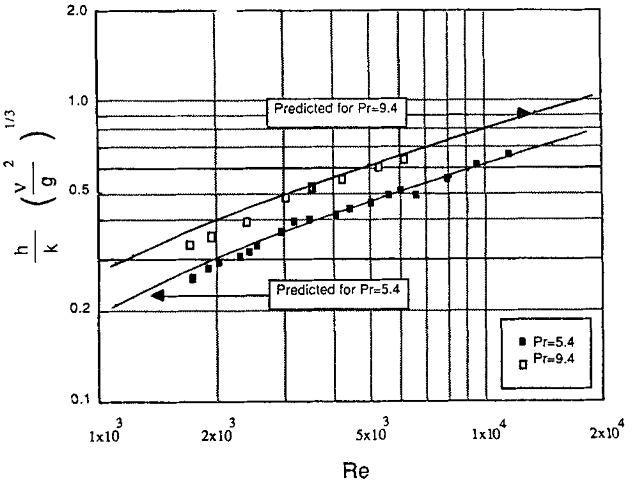

Figure 2. Variation of heat transfer coefficient with Reynolds number for heating.

Notation

b = coefficient, Eq. 18
B_1, B_2 = constant coefficients, Eq. 17
c_p = specific heat
C_1–C_4 = coefficients, Eq. 13
g = gravitational constant
h = heat transfer coefficient
k = thermal conductivity
K_3–K_5 = constant coefficients in series expansion for eddy diffusivity, Eq. 5

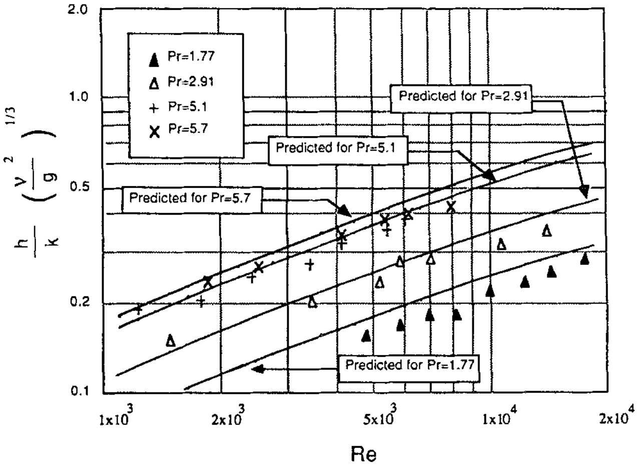

Figure 3. Variation of heat transfer coefficient with Reynolds number for evaporation.

K_{f3}–K_{f5} = constant coefficients in series expansion for corrected eddy diffusivity, Eq. 9
Ka = Kapitza number, $Ka = \mu^4 g/\rho\sigma^3$
M_1, M_2 = integration limits for y^+, Eq. 4
n = Reynolds number exponent, Eq. 20
P, Q = constant coefficients, Eq. 19
Pr = Prandtl number, $Pr = \nu/\alpha_t$
Pr_t = turbulent Prandtl number, $Pr_t = \epsilon/\epsilon_H$
q_0 = heat flux at wall
Re = Reynolds number, $Re = 4\,\Gamma/\rho\nu$
T = temperature
T_m = bulk temperature
T_{sat} = saturation temperature at interface
u = axial velocity
u^+ = dimensionless velocity, $u^+ = u/u^*$
u^* = friction velocity, $u^* = \sqrt{\tau_W/\rho}$
V_1, V_2 = constant coefficients, Eq. 6
x = coordinate in direction of flow
y = coordinate normal to flow
y^+ = dimensionless transverse direction, $y^+ = yu^*/\nu$

Greek letters

α_t = thermal diffusivity, $k/c_p\rho$
Γ = mass flow per wetted perimeter
δ = thickness of film
δ^+ = dimensionless thickness of film, $\delta^+ = \delta u^*/\nu$
ϵ_H = eddy diffusivity for heat
ϵ_m = eddy diffusivity for momentum
ϵ^+ = dimensionless eddy diffusivity, $\epsilon^+ = \epsilon_m/\nu$
ρ = density
σ = surface tension
τ_i = shear stress at interface
τ_W = shear stress at wall
ν = kinematic viscosity

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